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John Wilbur Tatom

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SHELL RADIATION

A THESIS

presented to

the Faculty of the Graduate Division

by

John Wilbur Tatom

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

Georgia Institute of Technology

May, 1960

SHELL RADIATION

Approved:

Date Approved by Chairman: May 30, 1960

ACKNOWLEDGMENT

I wish to thank my wife, Hilda, and also my friends at Chance Vought Aircraft Inc., Dallas, Texas, for their cooperation and assistance in the preparation of this thesis.

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SUMMARY

As we venture further into space, of major importance become the problems of both rejecting heat and controlling surface temperature by means of combined conduction and radiation heat transfer. While considerable effort is being made in this field, the analytical problems faced are severe due to the non-linear nature of the characteristic differential equations. The situation is further complicated due to the fact that the problems are often of the "boundary value" or "two point" type. Due to a general lack of information on this subject a study was promoted which involved writing and solving (analytically, where possible; numerically, elsewhere) the equations describing the steady-state temperature distribution along the surface of:

(a) A hollow sphere located in space and rotating about an axis oriented at a general angle with respect to the sun's rays

(b) an infinitely long non-rotating hollow cylinder also located in the solar radiation field

The equations include the effects of conduction, internal radiation, natural radiation and solar radiation. The main assumption is that the physical properties of the body remain constant. In general, both sets of equations take the form:

$$\frac{d^2\varphi}{d\theta^2} + C(\cot\theta)\frac{d\varphi}{d\theta} = K\left[\varphi^4 - A - BF(\theta)\right]$$

where:

φ = dimensionless temperature

Θ = angular position

$F(\Theta)$ = some function of Θ

A, B, C, K are constants representative of the various physical situations

Analytical solutions were obtained only for cases where $K = 0$ (thermal conductivity is infinite) and $K = \infty$ (thermal conductivity is zero). These solutions are

$$\varphi = \text{constant} \qquad K = 0$$
$$\varphi = \left[A + BF(\Theta) \right]^{1/4} \qquad K = \infty$$

Numerical solutions were necessary elsewhere. These solutions involved an iterative process, since the only two boundary conditions known ($\varphi'_{\Theta = 0} = \varphi'_{\Theta = \pi} = 0$) exist at two separate points. The equations have been solved on the IBM 704 for a large number of cases and are available in tabulated form.

The results demonstrate the strong effect of the internal radiation in damping out surface temperature variations. Thus for space vehicles having large open volumes in their interior, i.e., manned vehicles, the results indicate that skin temperature variations can be controlled easily by adjusting the ratio of the internal to the external surface emissivities. Also in certain practical interest cases, where the body's steady-state surface temperature is essentially constant, simple solutions to the transient problem are indicated; since in these cases the transient problem reduces to that of a point mass gaining or losing heat.

CHAPTER I

INTRODUCTION

Until recently, little or no attention has been given to the subject of radiation heat transfer to or from systems otherwise isolated. However with the advent of the "space age" the situation has changed. At present, considerable work is being done on such problems as the transient temperature variation of solid bodies gaining and losing heat by radiation at the surface, the temperature control of orbiting vehicles, and the temperature distribution along radiating surfaces. Several papers, by Abarbanel (1),* Wood and Carter (2), Wolfe (3), Lieblein (4), and Tatom (5 and 6) have been written which discuss these problems in detail. The close tie between these papers suggested to the author the following study.

Consider the problem of controlling the transient and steady-state temperature, and temperature variation along the surface of an interplanetary vehicle. In order to adequately solve this problem, we must first reach a thorough understanding of the vehicle's steady-state surface temperature distribution in absence of any other heat sources or sinks save that of the sun. Therefore in the present work, an analysis will be made of the steady-state temperature distribution along the surface of two simple, yet appropriate, hollow geometric bodies, located in

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*Numbers in parentheses refer to citations in the Bibliography.

interplanetary space, receiving energy from the sun, and radiating to infinity. Further, some conclusions will be reached regarding how to solve particular forms of the general problem of surface temperature control in space, both in the steady-state and transient situations.

CHAPTER II

DEVELOPMENT OF EQUATIONS

<u>(A) Hollow Sphere</u>.--In this section, we will consider the case of a hollow sphere located in the solar radiation field. To make the problem a little more general, let the sphere be rapidly rotating about an axis oriented at some angle Θ_s* with respect to the sun's rays (see Figures 1 and 2). From Figure 1, it appears that there are three general surface regions of interest:

(1) The region of continuous solar energy reception.

(2) The region of intermittent solar energy reception.

(3) The region of zero solar energy reception.

Hence, in any developed expression which describes the physical situation, it is certain that the terms involving the solar energy reception will have three different forms.

If the rotation is rapid enough, the time variations in local conditions are small and the temperature is essentially a function of angular position only. It follows then that the isotherms are concentric circles, since the temperature distribution is symmetric about the axis of rotation. Further, due to this symmetry and an assumed lack of any cusps at $\Theta = 0$ and $\Theta = \pi$, the slope of the temperature distribution curve will be zero at these points.

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*For a list of symbols see page 50.

In the development of the differential equation which describes the steady-state thermal behavior of the surface, an energy balance on an infinitesimal element must be performed. This balance involves equating the energy entering the element:

(1) Heat conducted from adjacent hotter elements.

(2) Heat absorbed from solar radiation.

(3) Heat absorbed from internal radiation.

to the energy leaving the element:

(1) Heat conducted to adjacent colder elements

(2) Heat radiated from both the inside and outside surfaces of the element.

Expressing this energy balance mathematically leads to the desired differential equation. But before writing the equation - in order to simplify matters - the following assumptions are made:

(a) All the physical properties of the surface material remain constant

(b) The radial temperature variation is neglible.

Also the dimensionless temperature $\varphi = T/T_\infty$, is introduced; where T_∞ is the temperature-in the case of zero surface conductivity-of an "isolated" element at $\Theta = 0$. Proceeding then, the equation is:

$$\frac{d^2\varphi_1}{d\Theta^2} + (\cot\Theta)\frac{d\varphi_1}{d\Theta} = K_1\left[\varphi_1^4 - A_1 - B_1 F_1(\Theta)\right] \qquad (1)$$

where:

$$K_1 = \frac{\sigma(\epsilon_i + \epsilon_e) R^2 T_{1\infty}^3}{kt}$$

$$A_1 = \frac{\epsilon_i/\epsilon_e}{4\cos\Theta_s + \epsilon_i/\epsilon_e}$$

$$B_1 = \frac{4}{4\cos\Theta_s + \epsilon_i/\epsilon_e}$$

$$F_1(\Theta)^* = \begin{cases} \cos\Theta_s \cos\Theta \text{ on } 0 \le \Theta \le \pi/2 - \Theta_s \\ \frac{1}{\pi}\left[\cos\Theta_s \cos\Theta \ \cos^{-1}(-\cot\Theta \ \cot\Theta_s) + \right. \\ \left. \sin\Theta_s \ \sin\Theta\sqrt{1-(\cot\Theta_s \ \cot\Theta)^2}\right] \text{ on } \frac{\pi}{2} - \Theta_s \le \Theta \le \\ \frac{\pi}{2} + \Theta_s \\ 0 \quad \text{on} \quad \frac{\pi}{2} + \Theta_s \le \Theta \le \pi \end{cases}$$

A detailed development of the above expressions can be found in Appendices A and B.

(B) Hollow Cylinder.--In this section, we will consider the case of an infinitely long, non-rotating, hollow cylinder located in the solar radiation field. Let the axis of the cylinder be oriented at an angle $(\frac{\pi}{2} - \beta_s)$ with respect to the sun's rays (see Figures 2 and 3). The similarity between this case and the preceding is apparent. Thus, again the temperature symmetry of before and the condition of zero derivative

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*In the calculations made for the sphere; $\Theta_s = 0$. Therefore:

$$F_1(\Theta) = \begin{cases} \cos\Theta & 0 \le \Theta \le \pi/2 \\ 0 & \pi/2 \le \Theta \le \pi \end{cases}$$

at $\Theta = 0$ and $\Theta = \pi$ is found. Further, in writing the differential equation, the same type of energy balance is performed, the same assumptions are made, and the dimensionless temperature φ is again utilized. However, this time a simplifying approximation is used to describe the internal energy reception. Continuing then, the desired equation is:

$$\frac{d^2\varphi_2}{d\Theta^2} = K_2\left[\varphi_2^4 - A_2 - B_2 F_2(\Theta)\right] \qquad (2)$$

where:

$$K_2 = \frac{\sigma(\epsilon_i + \epsilon_e) R^2 T_{2\infty}^3}{kt}$$

$$A_2 = \frac{\epsilon_i/\epsilon_e}{\pi + \epsilon_i/\epsilon_e}$$

$$B_2 = \frac{\pi}{\pi + \epsilon_i/\epsilon_e}$$

$$F_2(\Theta) = \begin{cases} \cos\Theta & \text{on} \quad 0 \le \Theta \le \pi/2 \\ 0 & \text{on} \quad \pi/2 \le \Theta \le \pi \end{cases}$$

A complete development of the above expressions, and a discussion and analysis of the discrepancies involved in the approximation:

$$A_2 = \frac{\epsilon_i/\epsilon_e}{\pi + \epsilon_i/\epsilon_e}$$

can be found in Appendices C and D respectively.

CHAPTER III

METHOD OF SOLUTION

It is apparent from the complexity of the equations in sections A and B of Chapter II that, in general, numerical solutions would be appropriate. However, in the two limiting cases of $K = 0$ and $K = \infty$, the problems can be handled analytically. When $K = 0$ (which corresponds to a case of infinite conductivity), the solution to both sets of equations is

$$\varphi_o = \text{constant} \neq f(\theta) = {}^{T_o}/_{T_\infty}$$

where: T_o is the constant radiating temperature required to reject all the energy absorbed from the sun.

When $K = \infty$ (which corresponds to a case of zero conductivity), the equations reduce to algebraic expressions. Thus:

$$\varphi_\infty = \left[A + B\,F(\theta) \right]^{1/4}$$

Therefore, in these two limiting conditions, the values of φ versus θ can be directly calculated.

In regard to the numerical solutions involved, particular forms of the above differential equations have been solved on the IBM 704 electronic computer using a standard routine.* The only input

*This routine is numbered 40-14-02 and would be available upon request through Chance Vought Aircraft, Incorporated, Dallas, Texas.

requirements using this routine are values of both the function φ and its first derivative, at $\Theta = 0$. Unfortunately the only boundary conditions known are that $\varphi'_{\Theta=0} = 0$ and $\varphi'_{\Theta=\pi} = 0$. Therefore, an iteration procedure was included in the final solution. Thus, several values of $\varphi_{\Theta=0}$ were guessed, $\varphi'_{\Theta=0}$ set to zero, and the equations solved in each case with the hope that the resulting values of $\varphi'_{\Theta=\pi}$ would be close to zero. Using the values of $\varphi_{\Theta=0}$ and the corresponding values of $\varphi'_{\Theta=\pi}$, an error curve was plotted and new values of $\varphi_{\Theta=0}$ were found which corresponded more closely to the condition that $\varphi'_{\Theta=\pi} = 0$. Then by progressive steps, the maximum difference in successive calculated values of φ was reduced to less than one part in ten thousand. The final result was a tabulation of φ versus Θ which met the boundary conditions to the desired degree of accuracy.

While this iteration process was relatively straightforward, the fact that so much time and energy was spent on it justifies a few comments:

(a) In general, the correct final solution is extremely sensitive to even the smallest error in $\varphi_{\Theta=0}$ and/or $\varphi'_{\Theta=\pi}$, the average number of iterations for each case being in the neighborhood of 14 to 15, in order to obtain four place accuracy.

(b) The function φ, for large values of K, has a very strong tendency to diverge, and therefore, good guesses of $\varphi_{\Theta=0}$ are required. In one notable case a discrepancy of 10^{-4} in the guessed value of $\varphi_{\Theta=0}$ resulted in a value of $\varphi'_{\Theta=\pi}$ greater than the capacity of the computing machine (10^{37}).

(c) Since the function $\mathcal{F}$, for good guesses of $\xi_{\theta=0}$, is well behaved; relatively large integration intervals can be used initially in the iteration process. However, bad guesses combined with large integration intervals can do more harm than good toward expediting matters.

CHAPTER IV

DISCUSSION OF RESULTS

The values of φ versus θ found in tables 1 through 5 and in Figure 4 correspond to the special case of $\theta_s = 0$ for the sphere, but tables 6 through 10 are completely general for the cylinder. The results are not surprising in that the temperature variations along the surface increase with increasing values of K. The existence of a cusp at $\theta = \pi/2$ for values of $K = \infty$ could also be expected, since in this case the temperature of elements on the hot side is a function of both location and the internal radiation, while abruptly at $\theta = \frac{\pi}{2}$ the temperature becomes a function of internal radiation only.

One significant and unexpected effect, however, is that of the internal radiation. It appears that as the ratio ϵ_i/ϵ_e is increased to values above 5 or 10, the temperature variations along the surface become vanishingly small. Thus large values of ϵ_i/ϵ_e have the same effect in damping out temperature variations as large values of the thermal conductivity.

CHAPTER V

CONCLUSIONS

The facts that:

(a) T_{∞} is a function of α_e/ϵ_e and ϵ_i/ϵ_e (see equations 4 and 6)

(b) Large values of the ratio ϵ_i/ϵ_e have the effect of damping out surface temperature variations

suggest a way to solve the original problem of controlling the temperature and temperature variations along the surface of space vehicles. Therefore for vehicles continuously in the sun and having large open volumes in their interior, i.e. manned vehicles; temperature control can be achieved simply and with a minimum weight penalty by adjusting the ratios α_e/ϵ_e and ϵ_i/ϵ_e. For example: a hollow nonrotating spherical vehicle having the following characteristics:

$t = .01$ feet	with:
$k = 200$ btu/hr ft $^{\circ}R$	$S = 425$ btu/hr ft^2
$\epsilon_i = .05$	$\sigma = .174 \times 10^{-8} \dfrac{\text{btu}}{\text{hr ft}^2\ {}^{\circ}R^4}$
$\epsilon_e = .01$	$T_{10} = 520^{\circ}R$
$\alpha_e = .012$	$T_{1\infty} = 575^{\circ}R$
$R = 5.5$ feet	$K = .3$

has a maximum temperature $T_{\theta = 0} = 536^{\circ}R$ and a minimum temperature, $T_{\theta = \pi} = 510^{\circ}R$. Probably a variation of only 26 degrees in this temperature range would be within the allowable tolerances. Even in cases where the temperature variations must be even smaller, i.e. where some form of artificial control is applied, these methods should be adequate for preliminary design calculations.

Further, in the case of large values of ϵ_i/ϵ_e which correspond to small variations in the surface temperature, the conclusions reached from the steady-state results for a hollow body remote from the earth can be applied to the case of a hollow earth satellite; since in this case, the transient problem arising out of the intermittent rate of energy reception reduces for practical purposes to that of a point mass gaining or losing heat. Therefore a single ordinary differential equation of the form:

$$\frac{dT_o}{d\gamma} = C_1\,Q(\gamma) - C_2T_o^4$$

where:

C_1 and C_2 are constants involving the weight, effective surface area, heat capacity, and surface emissivity of the vehicle,

$Q(\gamma)$ represents the combined effects of solar radiation, reflected energy and earth radiation,

T_o is the average radiating temperature,

can be written to describe the transient behavior of the vehicle. Due to the complex nature of $Q(\gamma)$, this equation can be handled probably most conveniently by numerical methods. Hence using such a procedure as outlined by Wolfe (3), such a transient problem can be handled fairly simply.

Therefore, for many problems of practical interest, where the surface temperature distribution must be known and under control, results have been presented and methods suggested which should lend themselves to rapid and relatively accurate calculations.

APPENDIX A

DEVELOPMENT OF DIFFERENTIAL EQUATIONS DESCRIBING THE THERMAL BEHAVIOR OF ELEMENTS ON THE SURFACE OF A HOLLOW ROTATING SPHERE LOCATED IN THE SOLAR RADIATION FIELD

APPENDIX A

Consider the case of the rapidly rotating hollow sphere located in the solar radiation field. The following expression governs the steady-state thermal behavior of any circular element

$$dq_{1x} + dq_{1g} + dq_{1z} = dq_{1u} + dq_{1v} \qquad (3)$$

where

dq_{1x} = Heat conducted from adjacent hotter elements

$$= -kA_c \frac{dT1}{ds}$$

$$= -k\,(2\pi R\, t \sin\Theta)\, \frac{dT1}{d\Theta}\, \frac{d\Theta}{ds}$$

$$= -2\pi k\, t \sin\Theta\, \frac{dT1}{d\Theta}$$

dq_{1y} = Heat absorbed from solar radiation

$$= S\alpha_e dA_r\, F_1(\Theta)^*$$

$$= S\alpha_e (2\pi R^2 \sin\Theta\, d\Theta) F_1(\Theta)$$

dq_{1z}** = Heat absorbed from internal radiation

$$= \sigma \epsilon_i (T_{10})^4\, dA_r$$

---

*See appendix B for detailed development of the expressions involved in $F_1(\Theta)$.

**It has been shown by Abarbanel (1) and Wood and Carter (2) that on the interior surface of a sphere the total energy received by each element from all other elements is constant, independent of the temperature distribution, and equal to $\sigma \epsilon_i (T_{10})^4 dA_r$. This is true for a sphere regardless of the internal surface emissivity ϵ_i.

$$= \sigma \epsilon_i (T_{10})^4 \; (2\pi R^2 \sin\theta \, d\theta)$$

Where T_{10} is the average radiating surface temperature and is defined by

$$\sigma \epsilon_e \pi D^2 (T_{10})^4 = S \alpha_e \frac{\pi D^2}{4}$$

$$T_{10} = \left[\frac{S \alpha_e}{4 \sigma \epsilon_e} \right]^{1/4} \qquad (4)$$

dq_{1u} = Heat conducted to adjacent colder elements

$$= dq_{1x} + \left[\frac{d}{ds} (dq_{1x}) \right] ds$$

$$= -2\pi \, kt \, \sin\theta \frac{dT_1}{d\theta} - 2\pi \, kt \left[(\cos\theta) \frac{dT_1}{d\theta} + (\sin\theta) \frac{d^2 T_1}{d\theta^2} \right] d\theta$$

dq_{1v} = Heat radiated from both exterior and interior surfaces of the element

$$= \sigma (\epsilon_e + \epsilon_i) T_1^{\,4} dA_r$$

$$= \sigma (\epsilon_e + \epsilon_i) T_1^{\,4} (2\pi R^2 \sin\theta \, d\theta)$$

Combining these expressions:

$$\frac{d^2 T_1}{d\theta^2} + \cot\theta \frac{dT_1}{d\theta} = \frac{R^2}{kt} \left[\sigma (\epsilon_e + \epsilon_i) T_1^{\,4} - \sigma \epsilon_i (T_{10})^4 - S \alpha_e F_1(\theta) \right] \qquad (5)$$

Then introducing the dimensionless temperature $\mathcal{T}_1 = T/T_{1\infty}$ where $T_{1\infty}$ is the temperature of an "isolated" element at $\theta = 0$ and is defined by

$$\sigma (\epsilon_e + \epsilon_i)(T_{1\infty})^4 = \sigma \epsilon_i (T_{10})^4 + S \alpha_e \cos\theta_s$$

$$T_{1\infty} = T_{10} \left[\frac{4\cos\theta_s + \epsilon_i/\epsilon_e}{1 + \epsilon_i/\epsilon_e} \right]^{1/4} \qquad (6)$$

The final equations are obtained:

$$\frac{d^2\varphi_1}{d\Theta^2} + \cot\Theta \, \frac{d\varphi_1}{d\Theta} = \frac{\sigma(\epsilon_e + \epsilon_i)R^2(T_{1\infty})^3}{kt} \left[\varphi_1^4 - \frac{\epsilon_i/\epsilon_e}{4\cos\Theta_s + \epsilon_i/\epsilon_e} - \frac{4}{4\cos\Theta_s + \epsilon_i/\epsilon_e} \cdot F_1(\Theta) \right] \tag{7}$$

APPENDIX B

ANALYSIS OF THE AVERAGE SOLAR ENERGY ABSORBED BY ELEMENTS ON THE SURFACE OF A HOLLOW ROTATING SPHERE LOCATED IN THE SOLAR RADIATION FIELD

APPENDIX B

In determining the average solar energy input for any elemental ring (dA_r) on the surface of the rotating sphere, the heat (d^2q_{1y}) absorbed by each subelement (d^2A_r) in the ring will be summated and then this total energy will be considered to be distributed evenly around the element. Thus the results will be good only for a rapidly rotating sphere. However, they can be applied to a stationary sphere, since rotation about an axis where $\Theta_s = 0$ is equivalent, so far as temperature variation is concerned, to no rotation at all.

In general, the solar energy input into each subelement will be:

$$d^2q_{1y} = (S\alpha_e \cos\omega) d^2A_r = (S\alpha_e \cos\omega)(R^2 \sin\Theta \, d\Theta \, d\phi)$$

where $\cos\omega$ is the direction cosine (see Figure 1).

Now from the paper by Wolfe (3)

$$\cos\omega = \cos\Theta \cos\Theta_s + \sin\Theta \sin\Theta_s \cos\phi$$

Therefore, for a circular element located in the region of continuous solar energy reception at an angle Θ

$$dq_{1y} = 2\, S\alpha_e R^2 \sin\Theta \, d\Theta \int_{\phi=0}^{\phi=\pi} (\cos\Theta \cos\Theta_s + \sin\Theta \sin\Theta_s \cos\phi)\, d\phi$$

$$= (2\pi R^2 \sin\Theta \, d\Theta) \underbrace{(S\alpha_e \cos\Theta \cos\Theta_s)}_{F_1(\Theta)} \quad \text{where} \quad 0 \le \Theta \le \frac{\pi}{2} - \Theta_s$$

In the region of intermittent solar energy reception, the equation describing the energy received by each subelement is the same as before, but in the summation process the limits of integration become a problem. However, again referring to the paper by Wolfe (3) it is seen that the "day-night" line is governed by the expression:

$$\cos\Theta\cos\Theta_s + \sin\Theta\sin\Theta_s\cos\phi = 0$$

$$\phi = \cos^{-1}\left[-\cot\Theta\cot\Theta_s\right] \qquad (11)$$

Therefore performing the same integration as before but using equation (11) as the upper limit:

$$dq_{1y} = (2\pi R^2\sin\Theta d\Theta)\ x \qquad (12)$$

$$\underbrace{\frac{S\alpha_e}{\pi}\left[\cos\Theta\cos\Theta_s\cos^{-1}(-\cot\Theta\cot\Theta_s)+\sin\Theta\sin\Theta_s\sqrt{1-(\cot\Theta\cot\Theta_s)^2}\right]}_{F_1(\Theta)}$$

where:

$$\frac{\pi}{2}-\Theta_s \leq \Theta \leq \frac{\pi}{2}+\Theta_s$$

In the region of no solar energy reception the term dq_{1y} is obviously zero.

APPENDIX C

DEVELOPMENT OF DIFFERENTIAL EQUATIONS DESCRIBING THE THERMAL BEHAVIOR OF ELEMENTS ON THE SURFACE OF AN INFINITELY LONG HOLLOW CYLINDER LOCATED IN THE SOLAR RADIATION FIELD

APPENDIX C

Consider the case of the infinitely long hollow cylinder located in the solar radiation field. The following expression governs the steady-state thermal behavior of any element:

$$dq_{2x} + dq_{2y} + dq_{1z} = dq_{2u} + dq_{2v} \tag{13}$$

where:

dq_{2x} = heat conducted from adjacent hotter elements

$$= -kA_c \frac{dT_2}{ds}$$

$$= -ktL \frac{dT_2}{d\Theta} \frac{d\Theta}{ds}$$

$$= -\frac{ktL}{R} \frac{dT_2}{d\Theta}$$

dq_{2y} = heat absorbed from solar radiation

$$= S\alpha_e \cos\beta_s dA_r F_2(\Theta)$$

$$= S\alpha_e \cos\beta_s LRd\Theta F_2(\Theta)$$

where

$$F_2(\Theta) = \begin{cases} \cos\Theta \text{ on } & 0 \le \Theta \le \pi/2 \\ 0 \text{ on } & \frac{\pi}{2} \le \Theta \le \pi \end{cases}$$

dq_{2z} = heat absorbed from internal radiation

$$= \sigma\epsilon_i (T_{20})^4 dA_r$$

$$= \sigma \epsilon_i (T_{20})^4 LRd\Theta \, *$$

where T_{20} is the average radiating temperature and defined by:

$$\sigma \epsilon_e \pi D T_{20}^4 = S \alpha_e D \cos \beta_s$$

$$T_{20} = \left[\frac{S\alpha_e \cos \beta_s}{\pi \sigma \epsilon_e} \right]^{1/4} \tag{14}$$

dq_{2u} = heat conducted to adjacent colder elements

$$= dq_{2x} + \left[\frac{d}{ds} (dq_{2x}) \right] ds$$

$$= -\frac{ktL}{R}\frac{dT_2}{d\Theta} - \left[\frac{ktL}{R}\frac{d}{d\Theta}\left(\frac{dT_2}{d\Theta}\right) \cdot \frac{d\Theta}{ds} \right] ds$$

$$= -\frac{ktL}{R}\frac{dT_2}{d\Theta} - \frac{ktL}{R}\frac{d^2T_2}{d\Theta^2} \, d\Theta$$

dq_{2v} = heat radiated from both exterior and interior surfaces of the element

$$= \sigma(\epsilon_e + \epsilon_i)T_2^4 \, dA_r$$

$$= \sigma(\epsilon_e + \epsilon_i)T_2^4 \, LRd\Theta$$

Combining these expressions:

$$\frac{d^2T_2}{d\Theta^2} = \frac{R^2}{kt}\left[\sigma(\epsilon_e + \epsilon_i)T_2^4 - \sigma\epsilon_i(T_{20})^4 - S\alpha_e \cos\beta_s \, F_2(\Theta) \right] \tag{15}$$

---

*As indicated previously this expression is only approximate, since the total energy received by each element from all other elements on the interior surface of a hollow cylinder is, in general, not constant, but dependent on location. The convenience of this simplification however, and the close degree of approximation afforded by it, justifies its use. See Appendix D for an evaulation of the discrepancy in F_2 involved in use of the above approximation.

Introducing $\varphi_2 = \frac{T_2}{T_{2\infty}}$ where $T_{2\infty}$ is defined by:

$$\sigma(\epsilon_e + \epsilon_i)(T_{2\infty})^4 = \sigma\epsilon_i(T_{20})^4 + S\alpha_e \cos\beta_s$$

$$T_{2\infty} = T_{20}\left[\frac{\pi + \epsilon_i/\epsilon_e}{1 + \epsilon_i/\epsilon_e}\right]^{1/4} \tag{16}$$

the equation simplifies to:

$$\frac{d^2\varphi_2}{d\Theta^2} = \frac{\sigma(\epsilon_i + \epsilon_e)R^2(T_{2\infty})^3}{kt}\left[\varphi_2^4 - \frac{\epsilon_i/\epsilon_e}{\pi + \epsilon_i/\epsilon_e} - \frac{\pi}{\pi + \epsilon_i/\epsilon_e}F_2(\Theta)\right] \tag{17}$$

APPENDIX D

ANALYSIS OF THE DISCREPANCIES INVOLVED IN THE ASSUMPTION THAT THE INTERNAL ENERGY RECEIVED PER UNIT AREA ON THE SURFACE OF AN INFINITELY LONG HOLLOW CYLINDER IS CONSTANT

APPENDIX D

In general, the total energy received by an infinitesimal area dA_N from a finite area A_M is defined by Jakob (7) (for a non uniform surface temperature) to be:

$$d\Gamma(d_N)_M = dA_N \int_{A_M} \sigma \epsilon_i T_{2M}^4 \frac{\cos\omega_N \cos\omega_M}{r^2} \, dA_M \qquad (18)$$

where:

$\cos\omega_M$ and $\cos\omega_N$ are direction cosines

r is the distance between elemental areas.

In determining expressions for the direction cosines for the two elements dA_M and dA_N located at (R, Θ_M, Z_M) and $(R, \Theta_N, 0)$ respectively, the answer comes easiest through the use of ordinary trigonometry and plane geometry. From Figure 3, it can be seen that:

$$\rho_M = \rho_N = \sqrt{R^2 + Z_M^2}$$

Now:

$$r = \sqrt{(X_M - X_N)^2 + (Y_M - Y_N)^2 + (Z_M - Z_N)^2}$$

$$= \sqrt{R^2(\cos\Theta_M - \cos\Theta_N)^2 + R^2(\sin\Theta_M - \sin\Theta_N)^2 + Z_M^2}$$

and from the law of cosines

$$\cos\omega_N = \cos\omega_M = \frac{r^2 + R^2 - \rho^2}{2rR}$$

$$= \frac{\left[1 - \cos\,(\Theta_M - \Theta_N)\right]}{\left[2 - 2\cos(\Theta_M - \Theta_N) + \left(\frac{Z_M}{R}\right)^2\right]^{1/2}} \tag{19}$$

Thus

$$d\Gamma(d_N)_M = \frac{2dA_N\,\sigma\,\epsilon_i}{\pi\,R}\int_{\Theta_N}^{\Theta_N+2\pi} T_{2M}^4\,d\Theta_M \int_0^{\infty}\frac{\left[1-\cos(\Theta_M-\Theta_N)\right]^2\,dZ_M}{\left[2-2\,\cos(\Theta_M-\Theta_N)+\left(\frac{Z_M}{R}\right)^2\right]^2}$$

$$\frac{d\Gamma(d_N)_M}{d\,A_N} = \frac{\sigma\,\epsilon_i}{2\sqrt{2}}\int_{\Theta_N}^{\Theta_N+2\pi} T_{2M}^4\left[1-\cos\,(\Theta_M-\Theta_N)\right]^{1/2}d\Theta_M \tag{20}$$

Again introducing

$$\varphi_2 = T_2/T_{2\infty}$$

$$\frac{d\Gamma(d_N)_M}{d\,A_N} = \frac{\sigma\epsilon_i\,T_{2\infty}^4}{2\sqrt{2}}\int_{\Theta_N}^{\Theta_N+2\pi}\varphi_{2M}^4\left[1-\cos(\Theta_M-\Theta_N)\right]^{1/2}d\Theta_M \tag{21}$$

Is some relation between φ_2 and Θ were known, equation (21) could be integrated and a measure could be obtained of how close the original assumption that

$$\frac{dq_{2z}}{dA_r} = \sigma\epsilon_i(T_{2\infty})^4(\varphi_{20})^4$$

really is. But the relation between φ_2 and Θ when $K = \infty$ is already known approximately, ie:

$$\varphi_2 = \left[A_2 + B_2F_2(\Theta)\right]^{1/4}$$

Further, if this relation is substituted in to equation (21) when $\Theta_N = 0$ and when $\Theta_N = \pi$, the resulting expressions correspond to the maximum possible deviations from the assumed conditions, (due to the temperature extremes). Therefore the ratios:

$$\left[\frac{d\,\Gamma_{(dn)M}\Big/dA_M}{dq_{2z}\Big/dA_r}\right]_{\Theta_N = 0} = \frac{1}{2\sqrt{2}}\;\frac{(\pi + \epsilon_i/\epsilon_e)}{(1+\epsilon_i/\epsilon_e)} \times \tag{22}$$

$$\left\{\int_0^{\pi/2} (A_2 + B_2\cos\Theta_M)(1-\cos\Theta_M)^{1/2}\,d\Theta_M + \int_{\pi/2}^{\pi} A_2(1-\cos\Theta_M)^{1/2}\,d\Theta_M\right\}$$

$$= \frac{.433766 + \epsilon_i/\epsilon_e}{1 + \epsilon_i/\epsilon_e}$$

and

$$\left[\frac{d\,\Gamma_{(dN)M}\Big/dA_M}{dq_{2z}\Big/dA_r}\right]_{\Theta_N = \pi} = \frac{1}{2\sqrt{2}}\;\frac{(\pi + \epsilon_i/\epsilon_e)}{(1+\epsilon_i/\epsilon_e)} \times \tag{23}$$

$$\left\{\int_0^{\pi/2} (A_2 + B_2\cos\Theta_M)(1+\cos\Theta_M)^{1/2}\,d\Theta_M + \int_{\pi/2}^{\pi} A_2(1+\cos\Theta_M)^{1/2}\,d\Theta_M\right\}$$

$$= \frac{1.48096 + \epsilon_i/\epsilon_e}{1 + \epsilon_i/\epsilon_e}$$

should be measures of the discrepancies involved in the assumption that the internal radiation is constant.

Now if the definitions are made that:

$$(T'_{2\infty})_{\Theta=0} = T_{20} \left\{ \frac{\epsilon_i/\epsilon_e \cdot \left[\frac{.433766 + \epsilon_i/\epsilon_e}{1+\epsilon_i/\epsilon_e}\right] + \pi}{1+\epsilon_i/\epsilon_e} \right\}^{1/4} \tag{24}$$

and

$$(T'_{2\infty})_{\Theta=\pi} = T_{20} \left\{ \frac{\epsilon_i/\epsilon_e \cdot \left[1.48096 + \epsilon_i/\epsilon_e\right]}{(1+\epsilon_i/\epsilon_e)^2} \right\}^{1/4} \tag{25}$$

and the ratios

$$\left(\frac{T'_{2\infty}}{T_{2\infty}}\right)_{\Theta=0} = \left(\frac{\xi'_{2\infty}}{\xi_{2\infty}}\right)_{\Theta=0} = \left\{ \frac{\epsilon_i/\epsilon_e \cdot \left[\frac{.433766 + \epsilon_i/\epsilon_e}{1+\epsilon_i/\epsilon_e}\right] + \pi}{\epsilon_i/\epsilon_e + \pi} \right\}^{1/4} \tag{26}$$

and

$$\left(\frac{T'_{2\infty}}{T_{2\infty}}\right)_{\Theta=\pi} = \left(\frac{\xi'_{2\infty}}{\xi_{2\infty}}\right)_{\Theta=\pi} = \left\{ \frac{1.48096 + \epsilon_i/\epsilon_e}{1+\epsilon_i/\epsilon_e} \right\}^{1/4} \tag{27}$$

are taken, the discrepancies in $\xi_{2\infty}$ involved in this assumption can be estimated. Figure 5 shows a plot of these last two ratios. Thus, while the discrepancies in the energy absorbed in the interior are relatively large, their effects on $\xi_{2\infty}$ are almost neglible, especially for values of ϵ_i/ϵ_e greater than 4 or 5. Further, since in general (for $K < \infty$) all other values of these ratios lie between the two curves shown, it seems the original assumption was not bad at all.

It can be argued qualitatively that the effect of reflected energy might further damp out the variations in the internal radiation. Therefore in physical cases where ϵ_1 is small, it may be that this effect plays a large role in maintaining the constancy of dq_{2z}.

APPENDIX E

ILLUSTRATIONS

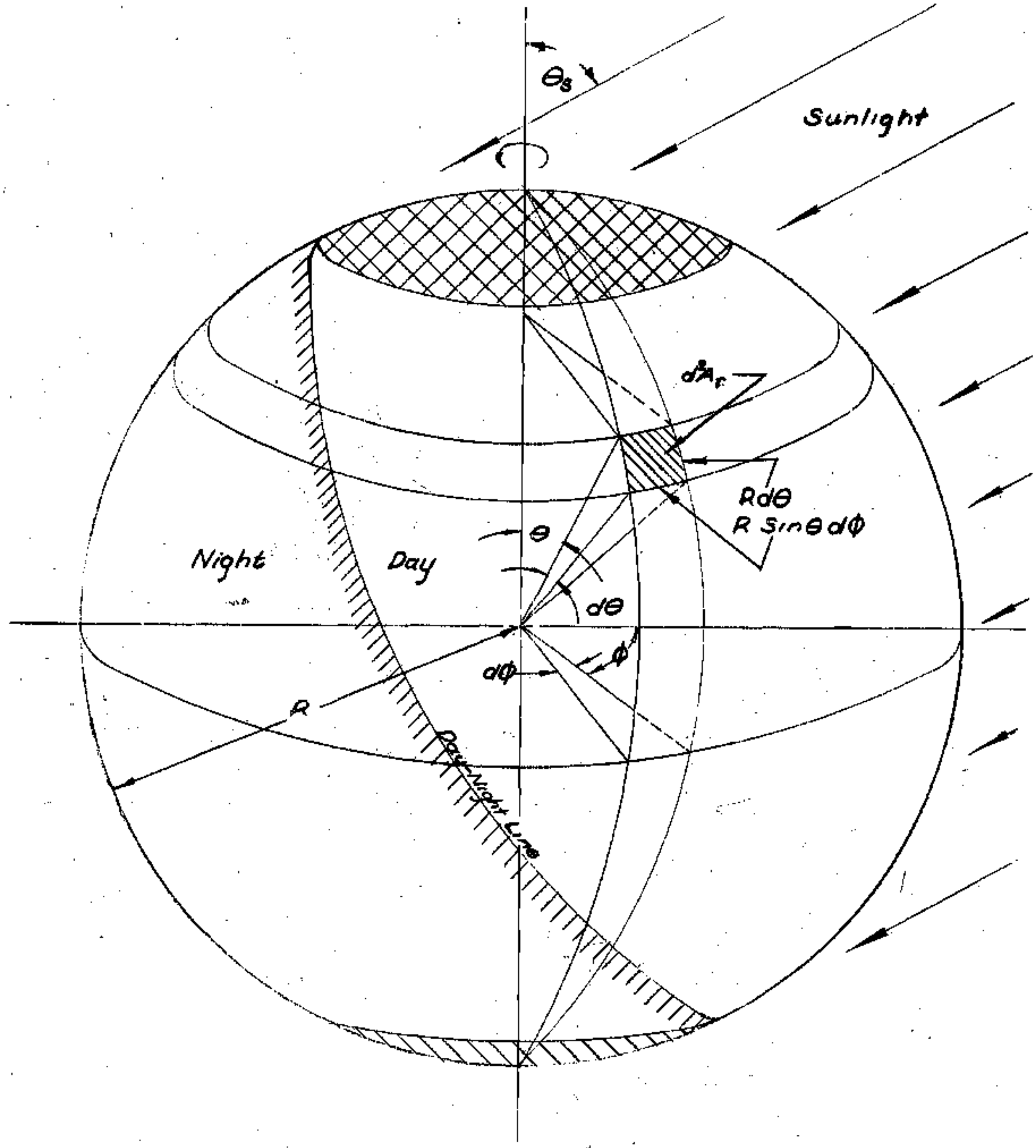

FIGURE 1

Reference Diagram For A Hollow Rotating Sphere Located In The Solar Radiation Field

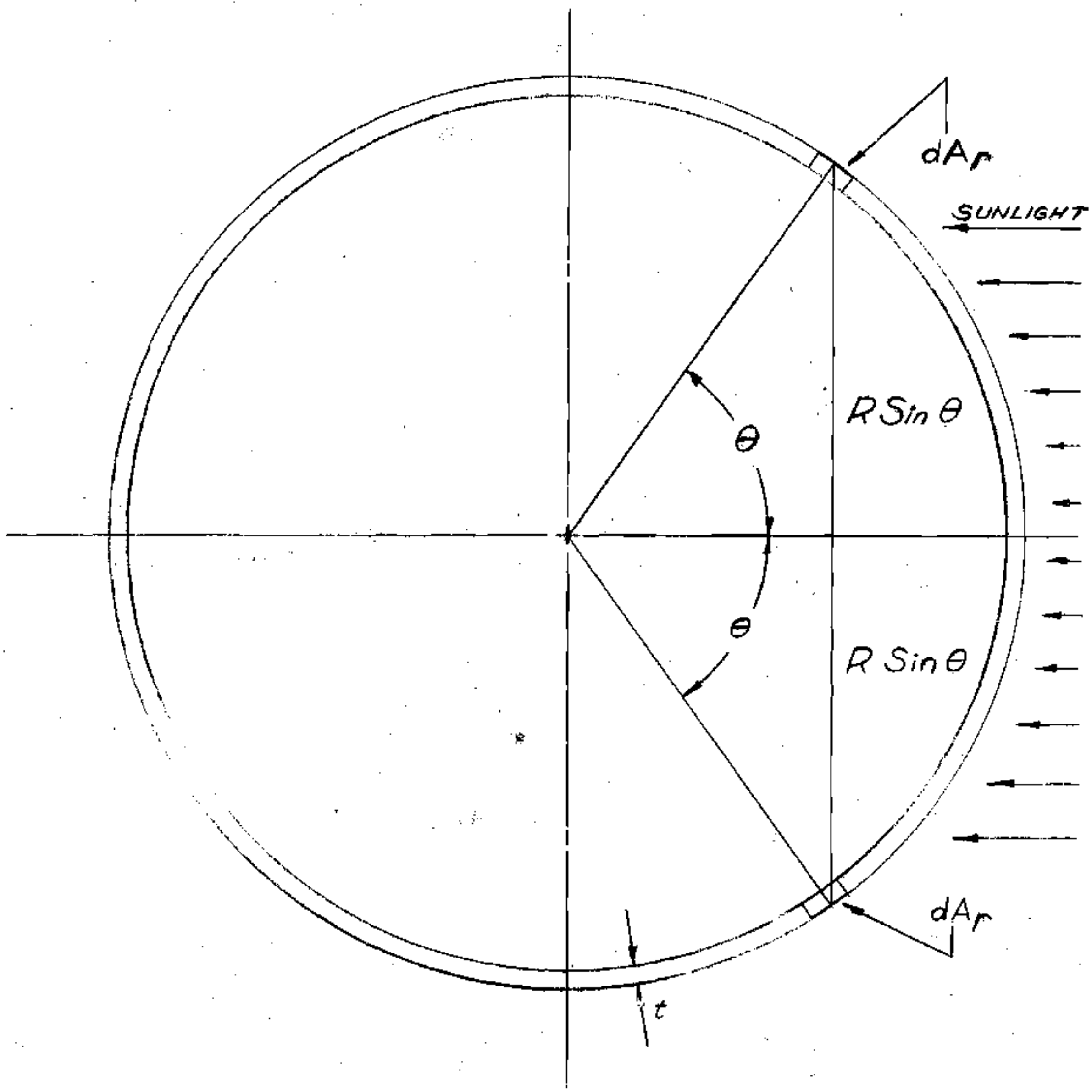

FIGURE 2

Reference Diagram For Rotating Sphere

(Θ_s = 0) And Infinite Cylinder

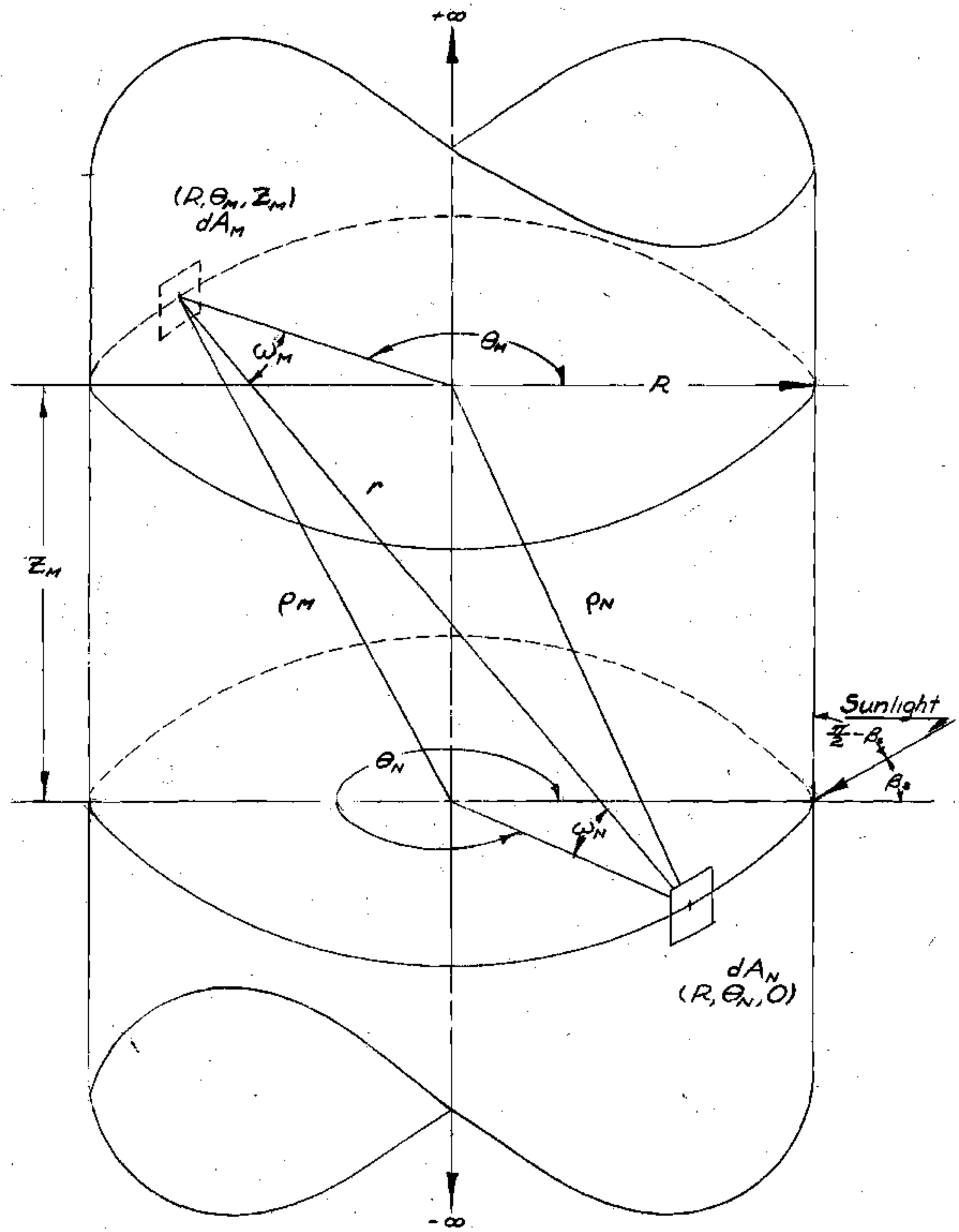

FIGURE 3

Reference Diagram For Infinitely Long Hollow Cylinder Located In The Solar Radiation Field

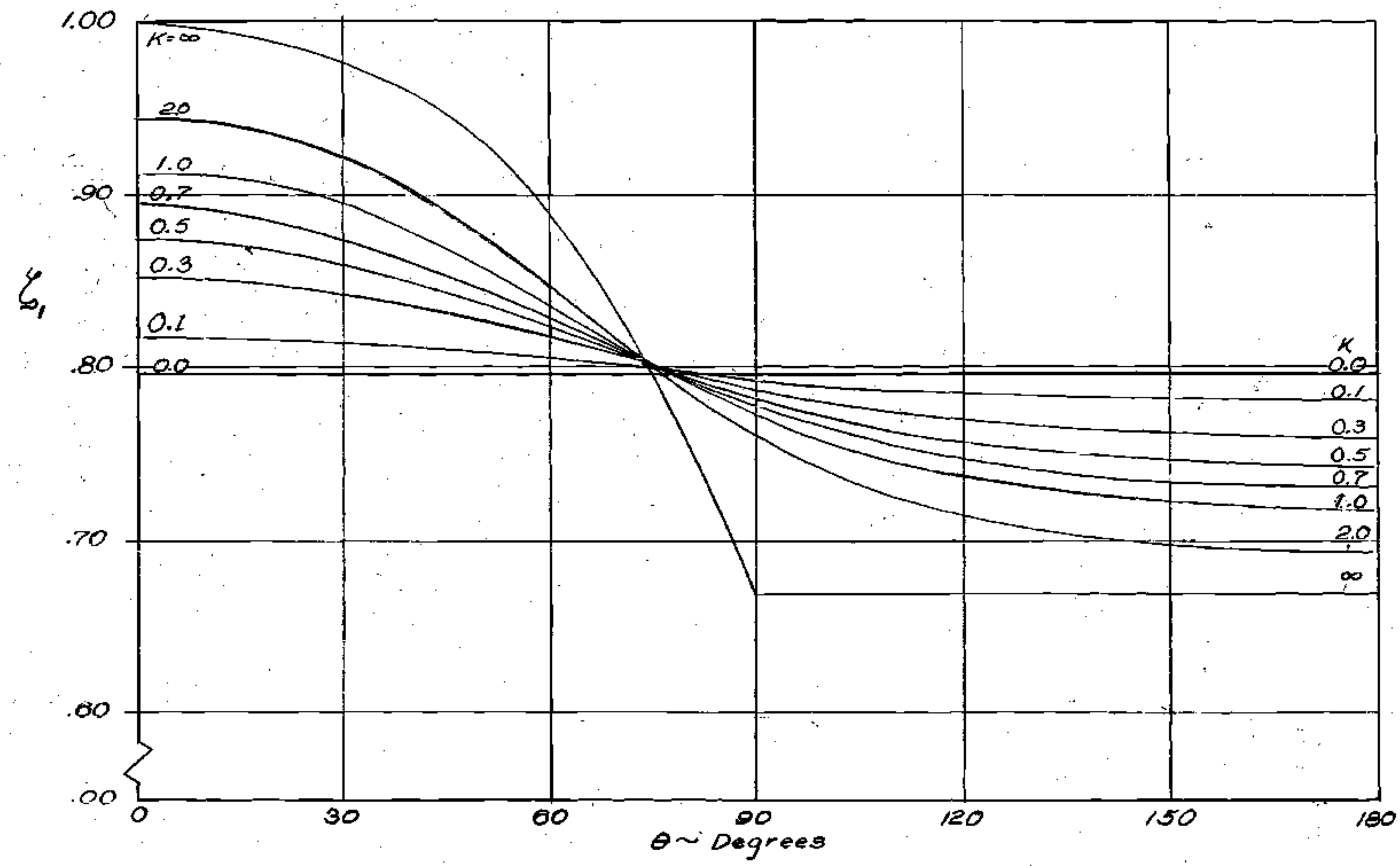

FIGURE 4

Dimensionless Temperature ζ_1 Versus Angular Position Θ, For A Sphere With $\epsilon_i/\epsilon_e = 1$, $\Theta_s = 0$

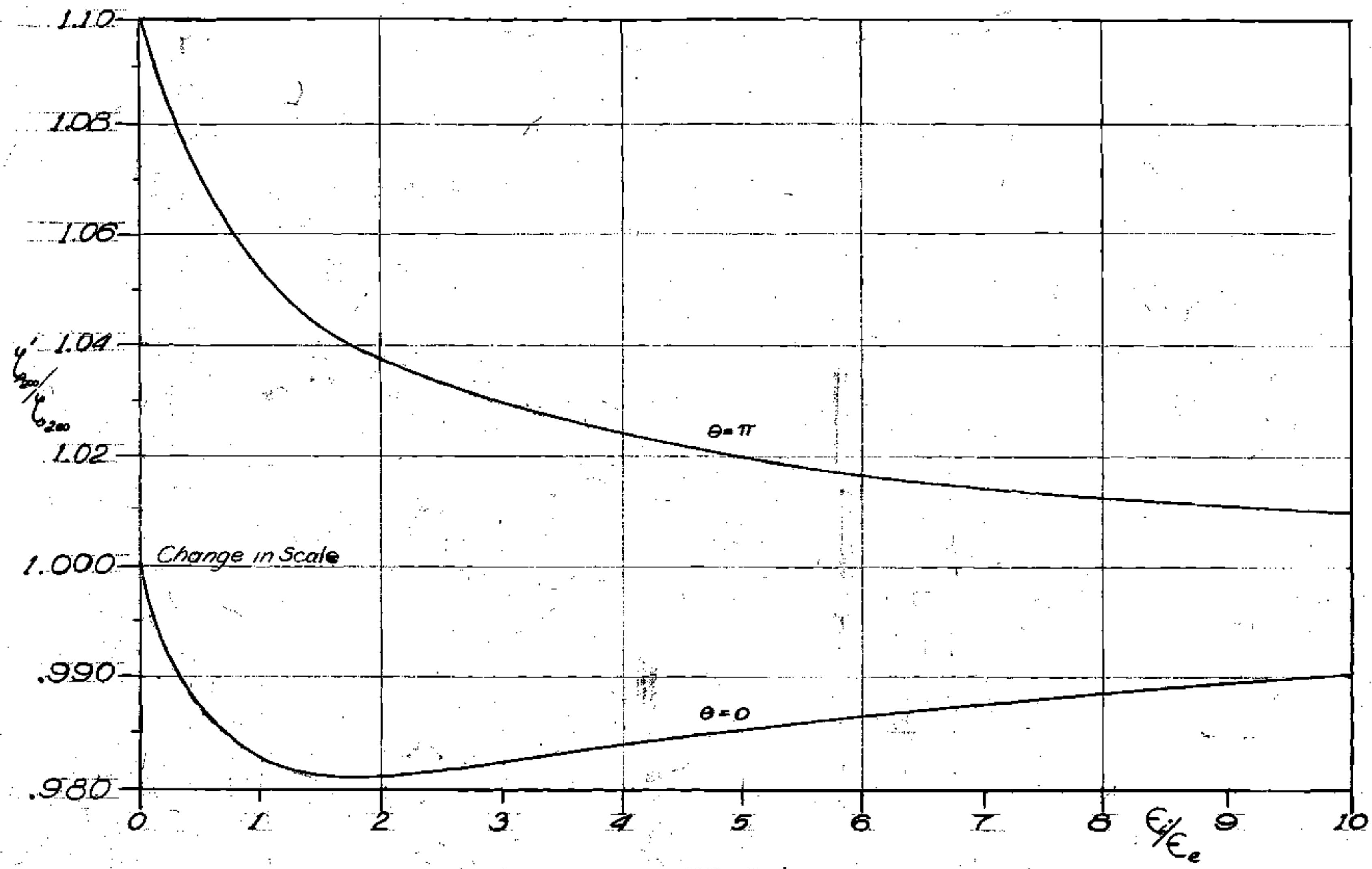

FIGURE 5

The Ratio $\varphi'_{2\infty}/\varphi_{2\infty}$ Versus ϵ_1/ϵ_e For Values Of $\Theta_N = 0$ And $\Theta_N = \pi$

APPENDIX F

TABLES

TABLE 1

Tabulation Of ξ_1 Versus Θ For Hollow Sphere

With $\Theta_s = 0$ And $\epsilon_i/\epsilon_e = 0$ ($A_1 = 0$, $B_1 = 1$)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	$K=\infty$
0	.7071	.7347	.7790	.8125	.8385	.8678	.9218	1.0000
10.2	.7071	.7342	.7775	.8103	.8357	.8644	.9174	.9960
20.2	.7071	.7326	.7732	.8039	.8277	.8545	.9046	.9843
30.2	.7071	.7300	.7663	.7936	.8146	.8385	.8835	.9642
40.2	.7071	.7265	.7571	.7798	.7972	.8169	.8544	.9349
50.2	.7071	.7224	.7460	.7632	.7761	.7906	.8181	.8945
60.2	.7071	.7178	.7336	.7446	.7524	.7609	.7759	.8396
70.0	.7071	.7131	.7209	.7254	.7280	.7300	.7310	.7647
80.0	.7071	.7083	.7081	.7061	.7033	.6987	.6849	.6455
90.0	.7071	.7039	.6965	.6886	.6810	.6704	.6430	0
100.0	.7071	.7002	.6867	.6741	.6626	.6475	.6100	0
110.0	.7071	.6971	.6787	.6623	.6479	.6293	.5846	0
120.0	.7071	.6946	.6721	.6528	.6361	.6149	.5649	0
130.0	.7071	.6925	.6669	.6453	.6268	.6036	.5497	0
140.0	.7071	.6909	.6628	.6394	.6195	.5948	.5382	0
150.0	.7071	.6896	.6597	.6349	.6141	.5883	.5297	0
160.0	.7071	.6888	.6575	.6319	.6104	.5839	.5239	0
170.0	.7071	.6882	.6562	.6300	.6082	.5812	.5205	0
180.0	.7071	.6881	.6558	.6294	.6075	.5804	.5194	0

TABLE 2

Tabulation Of ξ_i Versus Θ For Hollow Sphere

With $\Theta_s = 0$ And $\epsilon_i/\epsilon_e = 1$ ($A_1 = .2$, $B_1 = .8$)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=20	K= ∞
0	.7953	.8169	.8500	.8739	.8918	.9115	.9469	1.000
10.2	.7953	.8165	.8489	.8722	.8898	.9091	.9438	.9968
20.2	.7953	.8152	.8456	.8675	.8839	.9020	.9348	.9881
30.2	.7953	.8132	.8404	.8599	.8745	.8906	.9200	.9717
40.2	.7953	.8105	.8334	.8497	.8619	.8753	.8999	.9490
50.2	.7953	.8073	.8250	.8374	.8466	.8566	.8751	.9186
60.2	.7953	.8037	.8157	.8238	.8295	.8356	.8465	.8742
70.0	.7953	.8000	.8061	.8097	.8119	.8140	.8165	.8296
80.0	.7953	.7963	.7965	.7956	.7943	.7922	.7861	.7630
90.0	.7953	.7929	.7878	.7829	.7785	.7729	.7594	.6687
100.0	.7953	.7900	.7805	.7726	.7659	.7576	.7394	.6687
110.0	.7953	.7876	.7746	.7643	.7559	.7459	.7251	.6687
120.0	.7953	.7856	.7699	.7578	.7481	.7369	.7146	.6687
130.0	.7953	.7840	.7661	.7526	.7421	.7301	.7070	.6687
140.0	.7953	.7828	.7632	.7486	.7374	.7249	.7015	.6687
150.0	.7953	.7818	.7610	.7457	.7340	.7211	.6977	.6687
160.0	.7953	.7811	.7594	.7436	.7317	.7186	.6951	.6687
170.0	.7953	.7807	.7585	.7424	.7303	.7171	.6937	.6687
180.0	.7953	.7806	.7582	.7420	.7299	.7166	.6932	.6687

TABLE 3

Tabulation Of ζ_1 Versus Θ For Hollow Sphere

With Θ_s = 0 And ϵ_i/ϵ_e = 2 (A_1 = .33333, B_1 = .66667)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K= ∞
0	.8409	.8587	.8852	.9038	.9175	.9325	.9590	1.0000
10.2	.8409	.8584	.8843	.9025	.9159	.9305	.9566	.9974
20.2	.8409	.8573	.8816	.8987	.9113	.9250	.9495	.9896
30.2	.8409	.8557	.8774	.8926	.9038	.9160	.9380	.9765
40.2	.8409	.8535	.8718	.8845	.8938	.9039	.9223	.9581
50.2	.8409	.8508	.8650	.8747	.8817	.8893	.9030	.9337
60.2	.8409	.8478	.8575	.8638	.8683	.8729	.8811	.9029
70.0	.8409	.8448	.8498	.8526	.8544	.8561	.8581	.8656
80.0	.8409	.8417	.8420	.8415	.8406	.8392	.8351	.8186
90.0	.8409	.8389	.8350	.8315	.8283	.8243	.8151	.7598
100.0	.8409	.8366	.8292	.8234	.8186	.8128	.8006	.7598
110.0	.8409	.8346	.8246	.8170	.8110	.8042	.7906	.7598
120.0	.8409	.8330	.8209	.8120	.8052	.7976	.7835	.7598
130.0	.8409	.8317	.8179	.8081	.8007	.7927	.7786	.7598
140.0	.8409	.8307	.8156	.8051	.7974	.7891	.7751	.7598
150.0	.8409	.8299	.8139	.8028	.7949	.7865	.7727	.7598
160.0	.8409	.8294	.8127	.8013	.7932	.7847	.7712	.7598
170.0	.8409	.8291	.8120	.8004	.7922	.7837	.7703	.7598
180.0	.8409	.8290	.8118	.8001	.7919	.7834	.7700	.7598

TABLE 4

Tabulation Of φ_i Versus Θ For Hollow Sphere

With Θ_s = 0 And ϵ_i/ϵ_e = 5 (A = .55556, B = .44444)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K= ∞
0	.9036	.9153	.9318	.9431	.9512	.9599	.9753	1.0000
10.2	.9036	.9150	.9312	.9422	.9502	.9587	.9738	.9983
20.2	.9036	.9143	.9296	.9399	.9473	.9553	.9694	.9931
30.2	.9036	.9133	.9269	.9360	.9427	.9497	.9623	.9846
40.2	.9036	.9118	.9233	.9309	.9364	.9423	.9528	.9726
50.2	.9036	.9101	.9190	.9248	.9290	.9334	.9411	.9574
60.2	.9036	.9081	.9142	.9180	.9207	.9234	.9279	.9387
70.0	.9036	.9062	.9093	.9111	.9122	.9131	.9143	.9171
80.0	.9036	.9042	.9044	.9041	.9037	.9029	.9007	.8918
90.0	.9036	.9023	.9000	.8980	.8962	.8941	.8892	.8633
100.0	.9036	.9008	.8964	.8931	.8904	.8874	.8811	.8633
110.0	.9036	.8995	.8935	.8893	.8861	.8825	.8759	.8633
120.0	.9036	.8985	.8912	.8863	.8827	.8789	.8723	.8633
130.0	.9036	.8977	.8894	.8840	.8802	.8763	.8700	.8633
140.0	.9036	.8970	.8880	.8823	.8784	.8744	.8684	.8633
150.0	.9036	.8965	.8870	.8810	.8770	.8731	.8673	.8633
160.0	.9036	.8962	.8863	.8802	.8761	.8722	.8667	.8633
170.0	.9036	.8960	.8859	.8797	.8756	.8717	.8663	.8633
180.0	.9036	.8959	.8858	.8795	.8754	.8715	.8662	.8633

TABLE 5

Tabulation Of ξ_1 Versus Θ For Hollow Sphere

With $\Theta_s = 0$ And $\epsilon_i/\epsilon_e = 10$ (A = .71429, B = .28571)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K=∞
0	.9415	.9489	.9591	.9659	.9708	.9759	.9850	1.000
10.2	.9415	.9487	.9587	.9654	.9701	.9752	.9841	.9988
20.2	.9415	.9483	.9577	.9639	.9684	.9731	.9814	.9956
30.2	.9415	.9476	.9560	.9615	.9655	.9697	.9771	.9902
40.2	.9415	.9467	.9538	.9584	.9617	.9651	.9712	.9827
50.2	.9415	.9456	.9511	.9546	.9571	.9597	.9641	.9732
60.2	.9415	.9444	.9481	.9504	.9520	.9536	.9562	.9620
70.0	.9415	.9431	.9451	.9461	.9468	.9473	.9480	.9493
80.0	.9415	.9419	.9420	.9419	.9416	.9412	.9399	.9349
90.0	.9415	.9407	.9393	.9381	.9371	.9358	.9330	.9193
100.0	.9415	.9397	.9371	.9351	.9336	.9319	.9284	.9193
110.0	.9415	.9389	.9353	.9328	.9310	.9291	.9255	.9193
120.0	.9415	.9383	.9339	.9311	.9291	.9270	.9236	.9193
130.0	.9415	.9378	.9328	.9297	.9277	.9256	.9223	.9193
140.0	.9415	.9373	.9320	.9287	.9266	.9245	.9215	.9193
150.0	.9415	.9370	.9314	.9280	.9258	.9238	.9210	.9193
160.0	.9415	.9368	.9309	.9275	.9253	.9233	.9207	.9193
170.0	.9415	.9367	.9307	.9272	.9250	.9230	.9205	.9193
180.0	.9415	.9367	.9306	.9271	.9250	.9229	.9205	.9193

TABLE 6

Tabulation Of ξ_2 Versus Θ For Cylinder With

$\epsilon_i/\epsilon_e = 0$ ($A_2 = 0$, $B_2 = 1$)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0
0	.7511	.7968	.8527	.8854	.9070	.9282
10.2	.7511	.7958	.8504	.8824	.9034	.9241
20.2	.7511	.7931	.8440	.8736	.8931	.9123
30.2	.7511	.7888	.8337	.8594	.8763	.8930
40.2	.7511	.7830	.8198	.8404	.8537	.8668
50.2	.7511	.7760	.8032	.8174	.8262	.8345
60.2	.7511	.7682	.7845	.7914	.7949	.7976
70.0	.7511	.7600	.7651	.7645	.7624	.7588
80.0	.7511	.7518	.7454	.7370	.7291	.7188
90.0	.7511	.7439	.7269	.7113	.6981	.6818
100.0	.7511	.7369	.7107	.6892	.6716	.6505
110.0	.7511	.7308	.6968	.6704	.6495	.6247
120.0	.7511	.7256	.6851	.6548	.6312	.6036
130.0	.7511	.7212	.6754	.6420	.6162	.5865
140.0	.7511	.7176	.6676	.6317	.6044	.5730
150.0	.7511	.7149	.6616	.6239	.5954	.5628
160.0	.7511	.7129	.6574	.6184	.5891	.5557
170.0	.7511	.7117	.6549	.6152	.5853	.5514
180.0	.7511	.7113	.6540	.6141	.5841	.5500

TABLE 6 - Continued

Tabulation Of $\mathcal{E}_2$ Versus Θ For Cylinder With

$\epsilon_i / \epsilon_e = 0$ ($A_2 = 0$, $B_2 = 1$)

Θ	K=2	K=3	K=4	K=5	K=10	K=∞
0	.9611	.9744	.9813	.9854	.9932	1.000
10.2	.9565	.9697	.9766	.9808	.9890	.9960
20.2	.9428	.9558	.9629	.9673	.9762	.9843
30.2	.9202	.9324	.9395	.9441	.9543	.9642
40.2	.8886	.8992	.9057	.9103	.9216	.9349
50.2	.8485	.8559	.8610	.8648	.8759	.8945
60.2	.8011	.8033	.8053	.8072	.8147	.8396
70.0	.7497	.7447	.7419	.7403	.7384	.7647
80.0	.6959	.6822	.6729	.6661	.6481	.6455
90.0	.6459	.6237	.6081	.5961	.5598	0
100.0	.6048	.5767	.5567	.5412	.4937	0
110.0	.5719	.5399	.5171	.4996	.4461	0
120.0	.5456	.5109	.4863	.4675	.4108	0
130.0	.5248	.4881	.4625	.4429	.3843	0
140.0	.5085	.4706	.4442	.4241	.3645	0
150.0	.4964	.4576	.4307	.4103	.3501	0
160.0	.4880	.4486	.4214	.4008	.3402	0
170.0	.4830	.4433	.4159	.3952	.3345	0
180.0	.4814	.4416	.4141	.3934	.3327	0

TABLE 7

Tabulation Of ζ_z Versus Θ For Cylinder With

$\epsilon_i/\epsilon_e = 1$ (A_2 = .24145, B_2 = .75855)

Θ	K = 0	K = .1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K=∞
0	.8336	.8671	.9055	.9270	.9407	.9541	.9745	1.000
10.2	.8336	.8664	.9040	.9249	.9383	.9514	.9714	.9969
20.2	.9336	.8644	.8995	.9190	.9314	.9436	.9623	.9881
30.2	.8336	.8613	.8923	.9094	.9203	.9308	.9475	.9731
40.2	.8336	.8570	.8827	.8965	.9053	.9137	.9271	.9517
50.2	.8336	.8520	.8712	.8811	.8871	.8928	.9017	.9233
60.2	.8336	.8463	.8583	.8637	.8666	.8691	.8724	.8867
70.0	.8336	.8404	.8450	.8458	.8455	.8445	.8415	.8412
80.0	.8336	.8344	.8315	.8277	.8242	.8197	.8100	.7815
90.0	.8336	.8287	.8190	.8111	.8048	.7974	.7821	.7009
100.0	.8336	.8237	.8082	.7970	.7887	.7793	.7610	.7009
110.0	.8336	.8193	.7991	.7855	.7757	.7651	.7457	.7009
120.0	.8336	.8156	.7915	.7761	.7653	.7541	.7346	.7009
130.0	.8336	.8125	.7853	.7685	.7571	.7455	.7266	.7009
140.0	.8336	.8100	.7803	.7626	.7508	.7391	.7208	.7009
150.0	.8336	.8080	.7766	.7581	.7460	.7343	.7168	.7009
160.0	.8336	.8067	.7739	.7550	.7428	.7311	.7141	.7009
170.0	.8336	.8058	.7724	.7532	.7409	.7292	.7126	.7009
180.0	.8336	.8056	.7718	.7526	.7403	.7286	.7121	.7009

TABLE 8

Tabulation Of ξ_2 Versus Θ For Cylinder With

$\epsilon_i/\epsilon_e = 2$ (A_2 = .38898, B_2 = .61102)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K= ∞
0	.8740	.9004	.9297	.9457	.9558	.9656	.9806	1.000
10.2	.8740	.8999	.9285	.9441	.9540	.9636	.9783	.9976
20.2	.8740	.8983	.9251	.9396	.9487	.9576	.9713	.9905
30.2	.8740	.8959	.9195	.9322	.9402	.9480	.9600	.9787
40.2	.8740	.8925	.9121	.9225	.9289	.9350	.9446	.9618
50.2	.8740	.8885	.9033	.9107	.9151	.9193	.9256	.9398
60.2	.8740	.8841	.8934	.8975	.8997	.9016	.9039	.9122
70.0	.8740	.8794	.8832	.8840	.8839	.8833	.8811	.8794
80.0	.8740	.8747	.8729	.8703	.8680	.8650	.8583	.8388
90.0	.8740	.8703	.8634	.8579	.8537	.8487	.8384	.7898
100.0	.8740	.8663	.8552	.8476	.8420	.8358	.8239	.7898
110.0	.8740	.8629	.8484	.8392	.8328	.8260	.8138	.7898
120.0	.8740	.8600	.8428	.8324	.8255	.8185	.8068	.7898
130.0	.8740	.8576	.8382	.8271	.8199	.8128	.8020	.7898
140.0	.8740	.8557	.8346	.8229	.8156	.8086	.7986	.7898
150.0	.8740	.8542	.8318	.8198	.8124	.8056	.7964	.7898
160.0	.8740	.8531	.8299	.8177	.8103	.8036	.7949	.7898
170.0	.8740	.8525	.8288	.8164	.8090	.8024	.7941	.7898
180.0	.8740	.8523	.8284	.8160	.8086	.8021	.7938	.7898

TABLE 9

Tabulation Of ξ_2 Versus Θ For Cylinder With

$\epsilon_i/\epsilon_e = 5$ (A_2 = .61413, B_2 = .38587)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K=∞
0	.9265	.9428	.9599	.9690	.9747	.9802	.9886	1.0000
10.2	.9265	.9424	.9592	.9680	.9736	.9789	.9872	.9985
20.2	.9265	.9415	.9571	.9653	.9705	.9754	.9831	.9940
30.2	.9265	.9400	.9538	.9610	.9655	.9698	.9764	.9866
40.2	.9265	.9379	.9494	.9552	.9588	.9622	.9674	.9764
50.2	.9265	.9355	.9441	.9483	.9508	.9531	.9565	.9633
60.2	.9265	.9327	.9383	.9406	.9419	.9428	.9440	.9475
70.0	.9265	.9299	.9322	.9327	.9327	.9324	.9311	.9294
80.0	.9265	.9270	.9262	.9248	.9236	.9220	.9184	.9084
90.0	.9265	.9243	.9206	.9177	.9155	.9129	.9075	.8853
100.0	.9265	.9220	.9159	.9119	.9091	.9059	.9000	.8853
110.0	.9265	.9199	.9120	.9073	.9041	.9008	.8950	.8853
120.0	.9265	.9182	.9088	.9036	.9003	.8970	.8917	.8853
130.0	.9265	.9167	.9063	.9008	.8974	.8942	.8896	.8853
140.0	.9265	.9156	.9043	.8986	.8952	.8922	.8882	.8853
150.0	.9265	.9147	.9027	.8970	.8937	.8908	.8873	.8853
160.0	.9265	.9140	.9017	.8959	.8926	.8899	.8867	.8853
170.0	.9265	.9136	.9011	.8952	.8920	.8894	.8864	.8853
180.0	.9265	.9135	.9009	.8950	.8918	.8892	.8863	.8853

TABLE 10

Tabulation Of ρ_2 Versus Θ For Cylinder With

$\epsilon_i/\epsilon_e = 10$ ($A_2 = .76094$, $B_2 = .23906$)

Θ	K = 0	K=.1	K=.3	K=.5	K=.7	K=1.0	K=2.0	K=∞
0	.9565	.9664	.9765	.9818	.9851	.9883	.9932	1.000
10.2	.9565	.9662	.9761	.9812	.9845	.9876	.9924	.9991
20.2	.9565	.9656	.9749	.9796	.9826	.9855	.9899	.9963
30.2	.9565	.9647	.9729	.9771	.9796	.9821	.9859	.9918
40.2	.9565	.9634	.9702	.9736	.9757	.9776	.9806	.9856
50.2	.9565	.9620	.9671	.9695	.9709	.9722	.9741	.9778
60.2	.9565	.9603	.9636	.9649	.9656	.9662	.9668	.9684
70.0	.9565	.9586	.9600	.9603	.9602	.9600	.9593	.9582
80.0	.9565	.9568	.9564	.9556	.9549	.9540	.9519	.9465
90.0	.9565	.9552	.9531	.9514	.9502	.9487	.9457	.9340
100.0	.9565	.9538	.9503	.9481	.9465	.9447	.9415	.9340
110.0	.9565	.9525	.9480	.9454	.9437	.9419	.9388	.9340
120.0	.9565	.9515	.9462	.9433	.9415	.9398	.9371	.9340
130.0	.9565	.9506	.9447	.9417	.9399	.9383	.9360	.9340
140.0	.9565	.9499	.9435	.9405	.9388	.9372	.9353	.9340
150.0	.9565	.9494	.9427	.9396	.9379	.9365	.9348	.9340
160.0	.9565	.9490	.9421	.9390	.9374	.9360	.9346	.9340
170.0	.9565	.9488	.9417	.9387	.9371	.9358	.9345	.9340
180.0	.9565	.9487	.9416	.9386	.9370	.9357	.9344	.9340

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TABLE OF SYMBOLS

A	=	Constant in equations 1 and 2
B	=	Constant in equations 1 and 2
$\cos\omega$	=	Direction cosine
dA	=	Differential area, ft.^{2}
dq	=	Heat passing to or from elemental volume, $^{btu}/_{hr}$
$F(\Theta)$	=	Some function of the angle Θ
k	=	Conductivity, $^{btu}/_{hr\ ft\ ^\circ R}$
K	=	General constant defined in equations 1 and 2
R	=	Radius, feet
r	=	Distance between elemental areas, feet
S	=	Solar constant - 425 $\frac{btu}{hr\ ft^2}$
s	=	Arc length, feet
T	=	Temperature, $^\circ R$
t	=	Thickness, ft.
α	=	Solar absorbtivity
$\frac{\pi}{2}-\beta$	=	Angle between sun's rays and axis of infinite cylinder
$d\Gamma_{dN(M)}$	=	Heat flux received at element dA_N from area A_M, $^{btu}/_{hr}$
ϵ	=	Emissivity
ξ	=	Dimensionless temperature
Θ	=	Angular Position
ρ	=	Distance between the points (R, Θ_M, Z_M) and $(0,0,0)$ (see Figure 3)
σ	=	Stefan Boltzman Constant - $.174 \times 10^{-8}$ $\frac{btu}{hr ft^2 {}^\circ R^4}$
ϕ	=	Angular position (spherical coordinate system)

SUBSCRIPTS AND SUPERSCRIPTS

0	=	Average or mean radiating
1	=	Refers to hollow sphere
2	=	Refers to hollow cylinder
∞	=	Refers to conditions at $\theta = 0$ for $K = \infty$
x	=	Refers to energy entering an element by conduction
y	=	Refers to energy entering an element due to solar radiation
z	=	Refers to energy entering an element due to internal radiation
u	=	Refers to energy leaving an element by conduction
v	=	Refers to energy leaving an element by natural radiation
e	=	External
i	=	Internal
M	=	Refers to elemental area dA_M
N	=	Refers to elemental area dA_N
c	=	Refers to conduction heat transfer
r	=	Refers to radiation heat transfer
s	=	Refers to angular position relative to sun's rays
$'$	=	Refers to derived values of temperature; concerned with discrepancies involved in the assumption that A_2 is constant; also refers to differentiation with respect to θ